

MAS125 Calculus II 2006-2007

Problem sheet for exercise class 2

- **Please attend the exercise class to which you have been assigned**
- The instructor will present the starred problems in class.
- You should then work on the other problems on your own.
- The instructor and helper will be available for questions.
- Solutions will be available online after 5pm Friday.
- Many of the problems below involve using De Moivre's theorem. Simpler applications of this theorem can be found in this week's coursework, which complements the tutorial problems.

(*)Problem 1: Use De Moivre's theorem to obtain all the sixth roots of

$$32\sqrt{2}(1 - i)$$

and plot these roots on an Argand diagram.

(*)Problem 2: Use De Moivre's theorem to express the following complex number in polar form:

$$\frac{(\sqrt{3} + i)^2(1 - i)^3}{(i\sqrt{3} - 1)^3(i + 1)^2}.$$

Hint: first express the individual complex numbers in polar form. Can you see a simple way to evaluate this complex number without using De Moivre's theorem?

Problem 3: Use De Moivre's theorem to simplify as much as possible the expression

$$\frac{(\cos 3\theta + i \sin 3\theta)^6 (\cos 2\theta - i \sin 2\theta)^7}{(\cos 5\theta + i \sin 5\theta)^6 (\cos \theta - i \sin \theta)^8}.$$

Problem 4: Use De Moivre's Theorem to express $\cos 4\theta$ and $\sin 4\theta$ in terms of $\cos \theta$ and $\sin \theta$.

Problem 5: (More challenging.) Given the definition $\sinh x = (e^x - e^{-x})/2$, show that

$$\sinh^{-1} y = \ln(y + \sqrt{y^2 + 1}).$$

Use this to obtain the general solution of the equation $\sinh z = -2$ for complex z .
Hint: expand $\sinh z$ using the relations $\sinh(A + B) = \sinh A \cosh B + \cosh A \sinh B$, $\cosh iA = \cos A$ and $\sinh iA = i \sin A$. Then equate the real and imaginary parts.